Journal of Applied Mechanics and Technical Physics, Vol. 50, No. 1, pp. 1–8, 2009

NUMERICAL SIMULATION OF THE FLOW IN A VARIABLE-SECTION CHANNEL WITH PULSED-PERIODIC ENERGY SUPPLY

A. F. Latypov

UDC 533.6.011

Results of numerical simulations of a quasi-one-dimensional unsteady flow in a channel considered as an element of an air-breathing engine are presented. The influence of parameters of energy supplied in the pulsed-periodic mode (power, pulse frequency, and distribution of energy sources along the channel) on the characteristics of the flow with Mach numbers $\mathrm{M}_0 = 2.4$–4.0 *at the channel entrance is determined. A channel configuration that allows the energy supply distribution to be found from the condition of restriction of the maximum value of the gas temperature is proposed.*

Key words: *channel, pulsed-periodic energy supply, Euler equations.*

The idea of using an air-breathing engine in flight with hypersonic velocities (flight Mach numbers $\mathrm{M}_\infty \geqslant 6$) implies fuel burning in a supersonic air flow in the channel. The amount of the fuel being burned should be sufficient to obtain a required thrust (with the air-to-fuel ratio $\alpha \simeq 1$), and the flow has to be supersonic everywhere. Some schemes of fuel injection into the flow are proposed in [1, 2], and the structures of the supersonic flows formed thereby are given, though there is no information about implementation of these ideas. It is noted that the processes of mixing and combustion should be considered together. The problem of mixing of supersonic reacting gas flows requires an independent study. It is also necessary to solve the problem of the rational distribution of energy supply along the channel under the condition that the static temperature of the flow does not exceed a certain maximum value. This condition is imposed by the restriction on the degree of dissociation of combustion products, which decreases the gas-flow exergy.

The present paper describes the results of numerical simulations of an unsteady flow in a channel considered as an element of an air-breathing engine and consisting of segments with a constant section and an expanding section. The channel configuration is shown in Fig. 1 (x_1, x_2, x_3, and β are the varied parameters). In the classical scheme, energy supply in the combustion chamber is provided by burning the fuel in a certain polytropic process. In the case considered, the energy is supplied to the gas flow in a pulsed-periodic mode. Avoidance of mixing in considerations allows us to determine the direct effect of supplied energy parameters (power, pulse frequency, and distribution of energy sources along the channel) on flow characteristics. The flow Mach number at the channel entrance was varied in the range $\mathrm{M}_0 = 2.4$–4.0 corresponding to flight Mach numbers $\mathrm{M}_\infty = 6$–12. The parameters of the flow with energy supply were calculated by the unsteady quasi-one-dimensional Euler equations

$$\frac{\partial \boldsymbol{U}}{\partial t} + \frac{\partial \boldsymbol{F}}{\partial x} = \boldsymbol{G},$$

$$\boldsymbol{U} = (\rho y, \rho u y, e y), \qquad \boldsymbol{F} = (\rho u y, (p + \rho u^2) y, (p + e) u y),$$

$$\boldsymbol{G} = (0, p\, dy/dx, q y), \qquad p = (\gamma - 1)(e - \rho u^2/2),$$

Khristianovich Institute of Theoretical and Applied Mechanics, Siberian Division, Russian Academy of Sciences, Novosibirsk 630090; latypov@itam.nsc.ru. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, Vol. 50, No. 1, pp. 3–11, January–February, 2009. Original article submitted November 26, 2007.

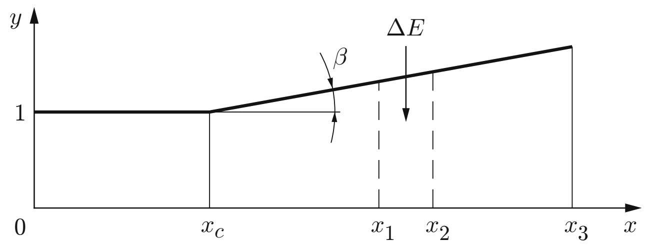

Fig. 1. Channel configuration: $x_c = 0.8$ is the length of the cylindrical part, x_3 is the total length of the channel, x_1–x_2 is the zone of energy supply, and β is the channel expansion angle.

TABLE 1

Flow Parameters at the Channel Entrance versus the Flight Mach Number

M_∞	M_0	T_0	γ	Q	θ
6	2.4	835	1.35	11.1	3.9
8	3.0	1050	1.33	8.8	2.0
10	3.6	1300	1.32	7.1	1.1
12	4.0	1610	1.30	5.8	0.7
16	4.5	2375	1.23	3.2	0.3

where all functions and parameters are dimensionless. The following quantities were chosen for normalization (the subscript 0 indicates the values at the channel entrance): channel height d_0, velocity of sound a_0, pressure p_0, and density $\rho^0 = p_0/a_0^2$. For dimensionless quantities, the previous notation is retained: $x = x/d_0$ is the streamwise coordinate, $y(x) = y(x)/d_0$ is the channel width, $t = ta_0/d_0$ is the time, $u = u/a_0$ is the velocity, $p = p/p_0$ is the pressure, $\rho = \rho/\rho^0$ is the density, $e = ea_0^2/\rho^0$ is the total energy density, $q = qd_0/(\rho^0 a_0^3)$ is the supplied energy power density, and γ is the ratio of specific heats.

In the case of pulsed-periodic energy supply, the value of q is determined from the expression

$$q = \Delta e(x) \sum_i \delta(t - i\Delta t)$$

where $\Delta e(x)$ is the supplied energy density, $\delta(t)$ is the pulsed Dirac function, and Δt is the energy supply period. The pulse duration is assumed to be so small that the change in gas density and velocity within the pulse can be neglected [3]. The free-stream conditions are set at the channel entrance, and linear extrapolation is used at the channel exit [4]. Between the instants of energy supply, the problem is solved by the fourth-order Maccormack method with artificial viscosity [5].

The measure of the supplied energy power is assumed to be Hu/L_0, which is the maximum power of energy released owing to hydrogen combustion in air (Hu is the caloric value of hydrogen and L_0 is the stoichiometric coefficient). In the expanding part of the channel, the energy supply is uniform in the x direction [$\Delta e(x)y = \mathrm{const}$]; the energy supplied in a given range $[x_1, x_2]$ during the period Δt is

$$\Delta E = \rho_0 u_0 y_0 z k \frac{\mathrm{Hu}}{L_0} \Delta t.$$

Here z is the channel width and k is the defined parameter (quantity inverse to the air-to-fuel ratio α). From the condition of identical energies, we obtain

$$\Delta e(x) y = \frac{\gamma \mathrm{M}_0 Q}{x_2 - x_1} \Delta t, \qquad Q = \frac{k\,\mathrm{Hu}}{L_0 a_0^2}.$$

The parameter

$$\theta = \frac{k\,\mathrm{Hu}/L_0}{u_0^2/2} = \frac{2Q}{\mathrm{M}_0^2}$$

determines the dimensionless amount of the supplied energy.

Table 1 contains the estimates of flow parameters (Mach number M_0, static temperature T_0, and ratio of specific heats γ) at the inlet exit (channel entrance) obtained by thermodynamic calculations versus the flight Mach number M_∞ (with a free-stream static temperature $T_\infty = 218$ K), and also the values of the energy supplied in the channel with $k = 1$.

In the case of pulsed-periodic energy supply into an initially supersonic flow in the channel, a complicated time-dependent system of shock waves, expansion waves, and contact discontinuities is formed and persists for a certain time. Then a periodic flow regime is established. This scenario of flow evolution was observed in all variants considered.

In a periodic flow, the gas parameters averaged over the period have steady-state values. The parameters at which pulsed energy supply occurs have also steady-state values. Therefore, the exergy method is applicable for the qualitative analysis of integral characteristics of the flow. The flow exergy σ is calculated with respect to flow parameters in the entrance cross section. The definition of exergy and the law of conservation of energy imply that

$$m_0\,\Delta t\,\sigma_3 = m_0\,\Delta t\,\sigma_0 + m_0\,\Delta t\,a_0^2 Q - (m_0 \Delta t\,\Delta S_1 + m_*\,\Delta S_2)T_0.$$

Here m_0 is the mass flow of the gas at the entrance, m_* is the gas mass in the energy supply zone, and ΔS_1 and ΔS_2 are the entropy increments in shock waves and during heat supply, respectively. Calculating the entropy increment during energy supply, we obtain the following expression for the dimensionless exergy $\bar{\sigma}_3 = \sigma_3/\sigma_0$:

$$\bar{\sigma}_3 = 1 + \theta - \frac{2\,\mathrm{Sh}}{\gamma(\gamma-1)\mathrm{M}_0^2}\int\limits_0^1 \frac{\ln(1+\eta)}{\bar{u}}\,d\xi - \frac{2\,\Delta\bar{S}_1}{\gamma \mathrm{M}_0^2}, \qquad \xi = \frac{x - x_1}{x_2 - x_1},$$

$$\eta(x) = \frac{\Delta T(x)}{T_*(x)} = \frac{\gamma(\gamma-1)Q\bar{u}(x)}{\mathrm{Sh}\,\bar{T}_*(x)}, \qquad \mathrm{Sh} = \frac{\Delta x}{u_0\,\Delta t}.$$

Here $\bar{u}(x) = u(x)/u_0$, $\bar{T}_*(x) = T_*(x)/T_0$, $T_*(x)$ is the distribution of the gas temperature in the zone $[x_1, x_2]$, Sh is the Strouchal number, $\Delta\bar{S}_1 = \Delta S_1/R$, and R is the gas constant. For $\mathrm{Sh} \gg 1$, which is a typical value for our problem, we obtain

$$\bar{\sigma}_3 = 1 + \theta\Big(1 - \int\limits_0^1 \frac{d\xi}{\bar{T}_*(x(\xi))}\Big) - \frac{2\Delta\bar{S}_1}{\gamma \mathrm{M}_0^2}.$$

In this case, gas heating is performed by small portions $[\eta(x) \ll 1]$; correspondingly, the pressure pulses are also small. In the opposite case, additional losses of exergy would occur in shock waves generated by large pressure pulses.

The effect of the problem parameters M_0, θ, Sh, x_1, x_2, x_3, and β on the structure of periodic flows being formed is illustrated below. We obtained the frequency of "saturation" of energy pulses above which the structure and the period-averaged distributions of parameters of the periodic flow remain unchanged. This frequency depends weakly on the problem parameters, and the period is equal to $\Delta t = 10^{-3}$. All calculations were performed for this value of Δt. The specific force f averaged over the period also becomes stabilized (Fig. 2):

$$f = \frac{[(p+\rho u^2)y]_3 - [(p+\rho u^2)y]_0}{(\rho u y)_0}.$$

If the energy supplied into the flow (with the initial Mach number $\mathrm{M}_0 = 2.4$) in the narrow zone in the cylindrical part of the channel corresponds to $k = 0.3$, a supersonic flow persists everywhere. The pressure and temperature increase in the energy-supply zone (curves 1 in Fig. 3). A similar character of the flow is observed in the case of supply of the same energy in the wide zone of the expanding part of the channel (curves 2). The values of exergy $\bar{\sigma}_3$ in variants 1 and 2, which are plotted by curves 1 and 2 in Fig. 3, are almost identical, but the length of the zone of elevated temperature in variant 2 is substantially smaller. An increase in energy to $k = 0.4$ leads to formation of a shock wave in the cylindrical part of the channel; the flow behind this shock wave in subsonic (curves 3). The shock wave performs low-amplitude oscillations with a frequency equal to the energy-supply frequency. A steady flow with a shock wave in the cylindrical part of the channel cannot be realized because of its instability. Two facts typical of steady flows should be noted: 1) in the case of energy supply, the pressure in a supersonic flow increases, and the pressure in a subsonic flow decreases; 2) in variant 3, which is plotted by curve 3 in Fig. 3, the flow structure is similar to that in the case of steady detonation; the only difference is the size of the characteristic regions. Curves 4 in Fig. 3 correspond to the flow in a channel with a large expansion angle

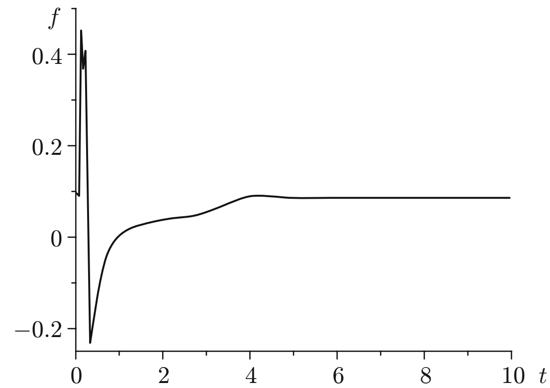

Fig. 2. Specific force averaged over the period versus time ($M_0 = 3.0$, $k = 0.75$, $\beta = 2^\circ$, $x_3 = 2$, $x_1 = 1.0$, and $x_2 = 1.6$).

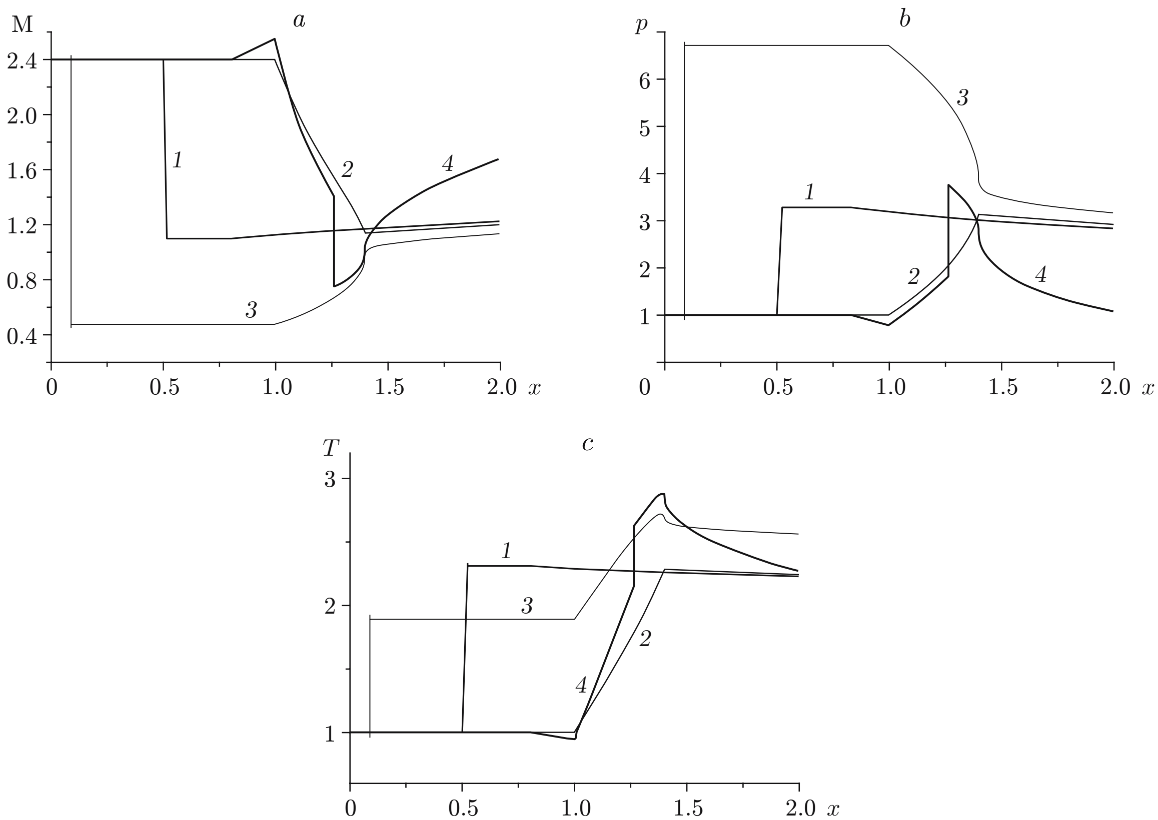

Fig. 3. Distributions of the Mach number (a), pressure (b), and temperature (c) along the channel ($M_0 = 2.4$ and $x_3 = 2$): 1) $k = 0.3$, $x_1 = 0.50$, $x_2 = 0.52$, and $\beta = 2^\circ$; 2) $k = 0.3$, $x_1 = 1.0$, $x_2 = 1.4$, and $\beta = 2^\circ$; 3) $k = 0.4$, $x_1 = 1.0$, $x_2 = 1.4$, and $\beta = 2^\circ$; 4) $k = 0.5$, $x_1 = 1.0$, $x_2 = 1.4$, and $\beta = 45^\circ$.

($\beta = 45^\circ$); a specific feature of this flow is the presence of a shock wave inside the energy-supply zone. The flow velocity behind this shock wave is subsonic, and the Mach number at the end of this zone is $M = 1$. Such a flow is not formed in the case of a small expansion angle ($\beta = 2^\circ$): for $k = 0.39$, the flow structure is similar to that in variant 2; for $k = 0.40$, a flow with a shock wave outside the energy-supply zone is formed (variant 3).

Let us consider a one-dimensional steady flow in a channel with a variable cross section with heat addition and in the presence of some processes of energy dissipation. Let us define a function $H(x)$ determining the amount of heat added between the entrance cross section Ω_0 and the cross section $\Omega(x)$ and a specific force $j_d(x)$ corresponding

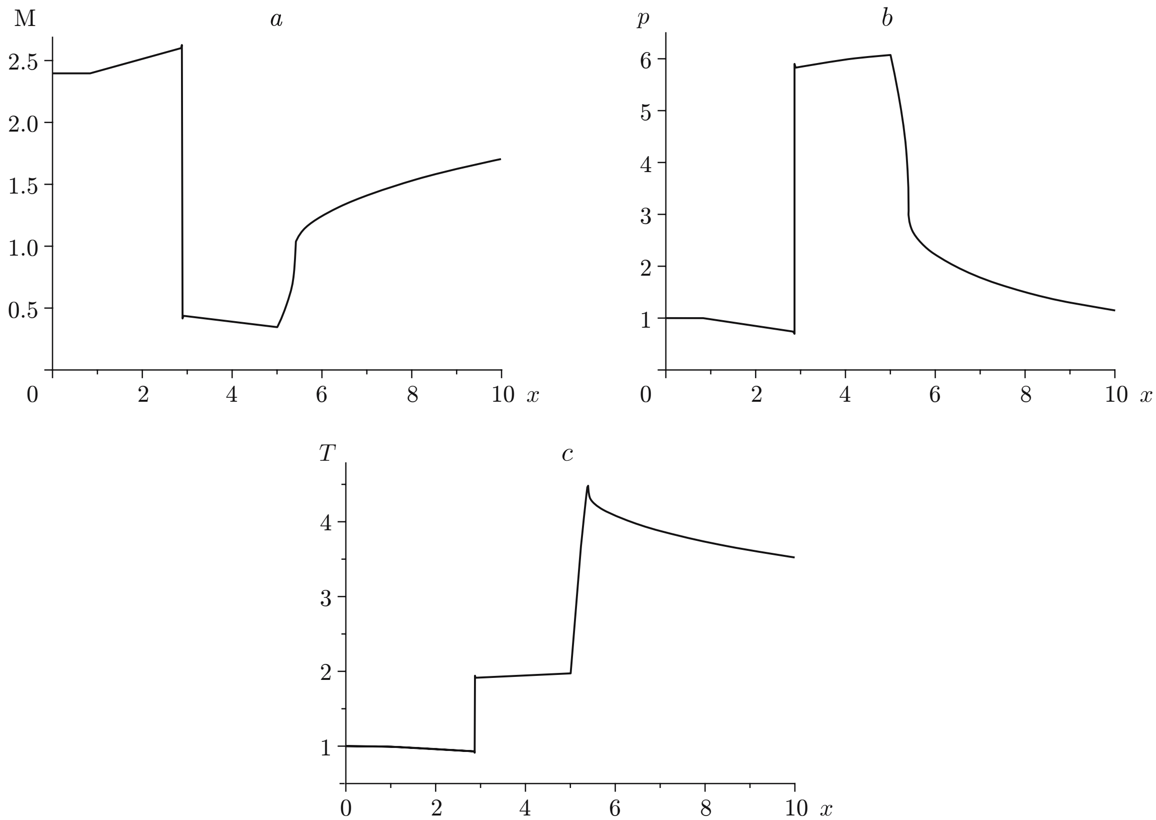

Fig. 4. Distributions of the Mach number (a), pressure (b), and temperature (c) along the channel ($\mathrm{M}_0 = 2.4$, $k = 1$, $\beta = 6^\circ$, $x_3 = 10$, $x_1 = 5.0$, and $x_2 = 5.4$).

to the energy-dissipation process; $H_d(x) = j_d(x)\, d_0$ is a function of energy dissipation. Let us write the conservation laws for an elementary volume dx:

$$\Big(1 + \frac{\gamma-1}{2}\,\mathrm{M}^2\Big)\frac{dT}{T} + (\gamma-1)\mathrm{M}^2\,\frac{d\mathrm{M}}{\mathrm{M}} = \frac{dH(x)}{c_pT}, \qquad \frac{dp}{p} - \frac{1}{2}\,\frac{dT}{T} + \frac{d\mathrm{M}}{\mathrm{M}} = -\frac{d\Omega(x)}{\Omega(x)},$$
$$\frac{dp}{p} + \frac{1}{2}\,\gamma\mathrm{M}^2\,\frac{dT}{T} + \gamma\mathrm{M}^2\,\frac{d\mathrm{M}}{\mathrm{M}} = -\frac{H_d(x)\,dx}{RT}. \tag{1}$$

The determinant of this system is

$$\Delta_0 = \begin{vmatrix} 0 & 1+(\gamma-1)\mathrm{M}^2/2 & (\gamma-1)\mathrm{M}^2 \\ 1 & -1/2 & 1 \\ 1 & \gamma\mathrm{M}^2/2 & \gamma\mathrm{M}^2 \end{vmatrix} = 1 - \mathrm{M}^2$$

and vanishes at $\mathrm{M} = 1$. For the continuous solution to be extended to the point $x = x_*$ where $\mathrm{M}(x_*) = 1$, the determinant obtained by replacing some column in the determinant Δ_0 by the vector column of the right side of the equations should also be equal to zero. The resultant condition has the form

$$\frac{H'(x_*)}{c_pT} + \frac{H_d(x_*)}{RT} - \frac{\Omega'(x_*)}{\Omega(x_*)} = 0 \tag{2}$$

(the prime indicates the derivative of the function with respect to x). This relation determines the structure of periodic flows formed at high Strouchal numbers. In this problem, we have $H_d(x) = 0$; as $\Omega'(x) > 0$ in the expanding section, the cross section $\Omega(x_*)$ is always located inside the energy-supply zone (normally, $x_* = x_2 - \varepsilon$ and $\varepsilon \sim 0$). The flow behind this cross section can be both supersonic and subsonic. The conditions of formation of different flow structures in variant 4 (see Fig. 3) are not clear.

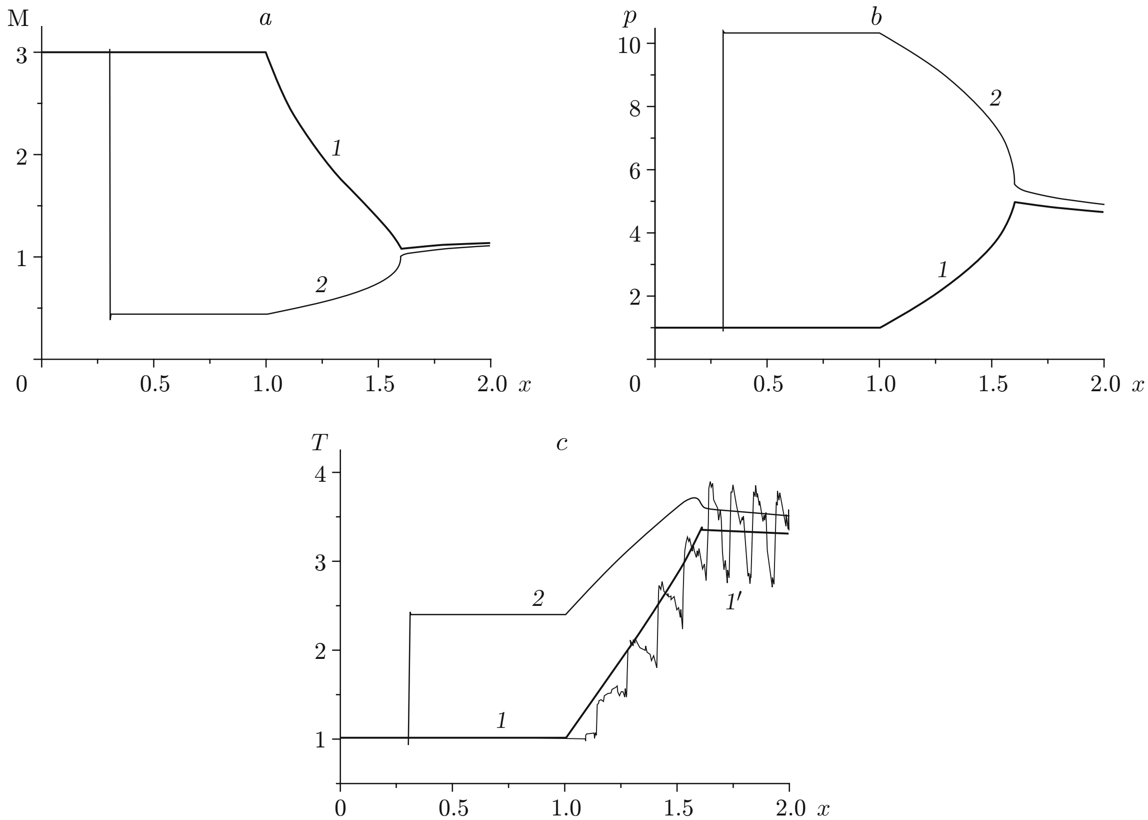

Fig. 5. Distributions of the Mach number (a), pressure (b), and temperature (c) along the channel ($\mathrm{M}_0 = 3.0$, $\beta = 2^\circ$, $x_3 = 2$, $x_1 = 1.0$, and $x_2 = 1.6$): curve 1 refers to $k = 0.75$ and $\Delta t = 10^{-3}$; curve 2 refers to $k = 0.80$ and $\Delta t = 10^{-3}$; curve $1'$ refers to $k = 0.75$ and $\Delta t = 5 \cdot 10^{-2}$.

The results calculated for the maximum energy in the accepted measure $k = 1$ are plotted in Fig. 4. In this case, the shock wave is also formed, but this happens in the expanding part of the channel. The energy is supplied into a subsonic flow. The Mach number in a small vicinity of the right boundary of the energy-supply zone is also equal to unity. After that, the flow acquires a supersonic velocity.

Figure 5 shows the distributions of parameters along the channel for the Mach number at the channel entrance $\mathrm{M}_0 = 3$ and $\Delta t = 10^{-3}$ for $k = 0.75$ (curve 1) and $k = 0.80$ (curve 2). For $k = 0.75$, the flow velocity is supersonic everywhere; for $k = 0.80$, the energy is supplied into a subsonic flow. The shock wave weakly fluctuating with respect to x is located in the cylindrical part of the channel. Figure 5c shows the effect of the increase in the energy-supply period to $\Delta t = 5 \cdot 10^{-2}$ on the distribution of flow parameters [this figure shows only the dependence (curve $1'$); the dependences $\mathrm{M}(x)$, $p(x)$, and $\rho(x)$ behave similarly]. A flow with parameters oscillating with a significant amplitude is formed. At $\Delta t = 10^{-3}$, the Strouchal number is $\mathrm{Sh} = 2 \cdot 10^2$. In the energy-supply zone, the gas particle experiences multiple [in small portions, i.e., $\eta(x) \ll 1$] energy actions (curve 1 in Fig. 5c). At $\Delta t = 5 \cdot 10^{-2}$, the Strouchal number is substantially smaller ($\mathrm{Sh} = 4$); as the energy supplied is the same, the increase in temperature under a single action is significantly greater. This is the reason for substantial fluctuations of flow parameters observed. The specific force averaged over the period is smaller than in the case with $\Delta t = 10^{-3}$.

An increase in the channel length ($x_3 = 10$) and expansion angle ($\beta = 6^\circ$) allows a greater amount of energy ($k = 0.85$) to be supplied with the supersonic flow velocity being retained. The distributions of the parameters M, p, and T along the channel are qualitatively identical to those in variant 1, which is illustrated by curves 1 in Fig. 5. In the case of addition of the maximum energy ($k = 1$) in the narrow zone in a channel of length $x_3 = 10$ with an expansion angle $\beta = 2^\circ$, a flow with a subsonic portion is formed. The shock wave is located in the expanding part of the channel (Fig. 6). The flow in the energy-supply zone is similar to the flow in the case of steady detonation.

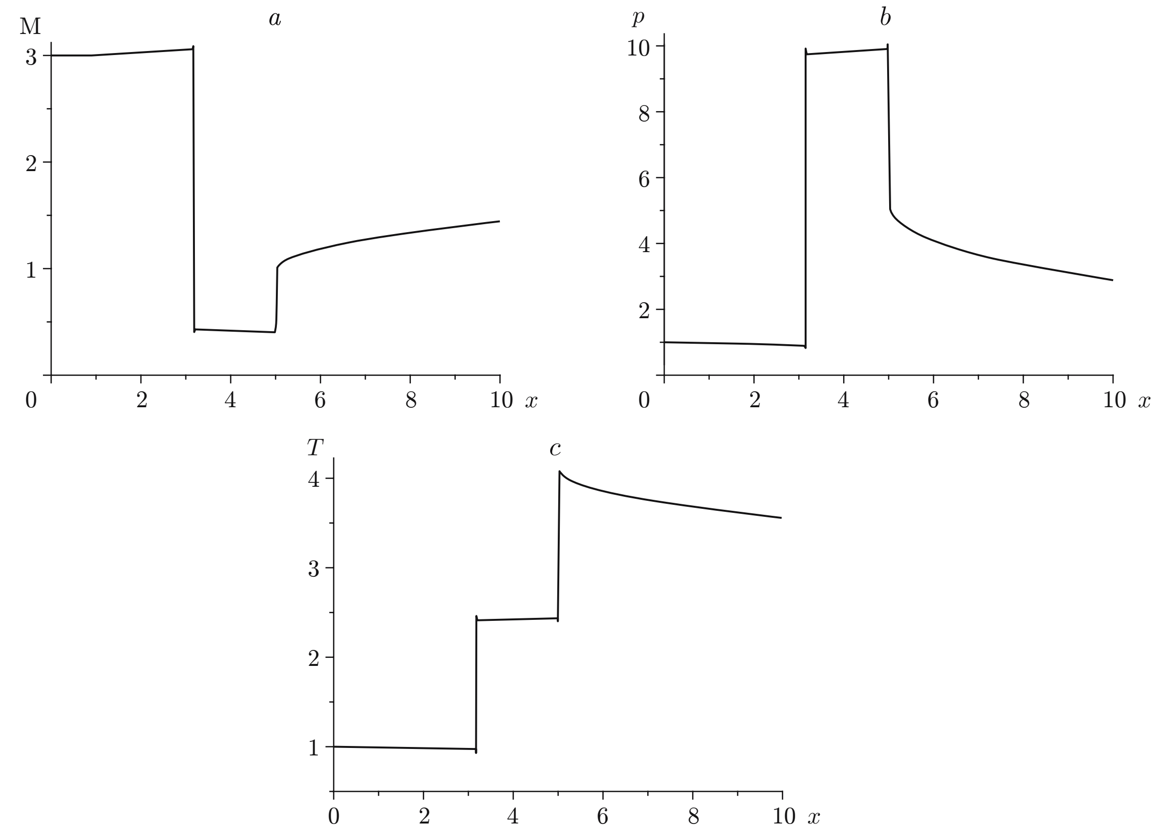

Fig. 6. Distributions of the Mach number (a), pressure (b), and temperature (c) along the channel with energy supply in the narrow zone ($M_0 = 3$, $k = 1.0$, $\beta = 2^\circ$, $x_3 = 10$, $x_1 = 5.00$, and $x_2 = 5.04$).

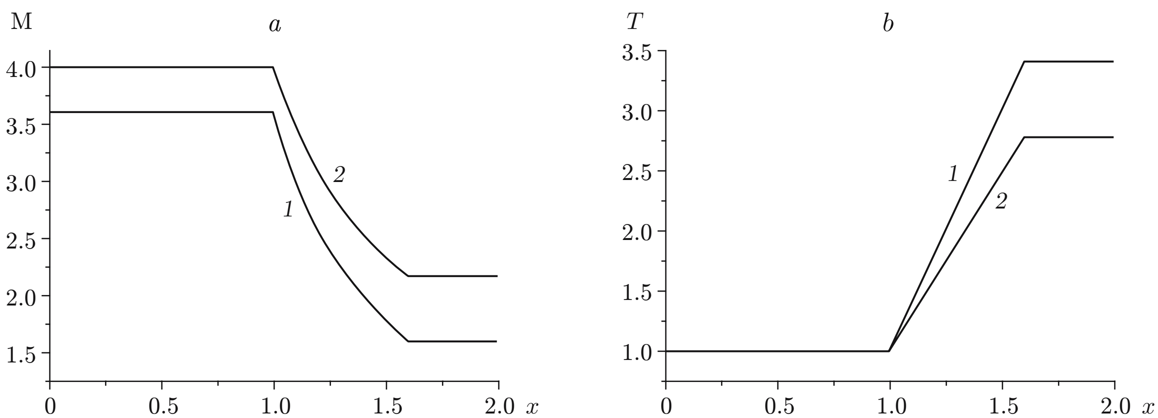

Fig. 7. Distributions of the Mach number (a) and temperature (b) along the channel ($k = 1.0$, $\beta = 2^\circ$, $x_3 = 2$, $x_1 = 1.0$, and $x_2 = 1.6$) for $M_0 = 3.6$ (1) and 4.0 (2).

At $M_0 = 3.6$ and 4.0, it is possible to supply the maximum energy with a supersonic velocity of the flow preserved everywhere (Fig. 7).

At flight Mach numbers $M_\infty \geqslant 8$, it is necessary to monitor the maximum gas temperature $T_{\max}$. For the fuel energy to be effectively used, this value should be approximately 3000 K. The results presented show that the gas temperature reaches high values in the case of hypersonic flight velocities even if a supersonic flow velocity is ensured in the energy-supply zone. To satisfy the condition $T \leqslant T_{\max}$, it is necessary to change the channel configuration so that the energy is supplied in several isolated zones (sections) divided by expanding parts, with a supersonic flow

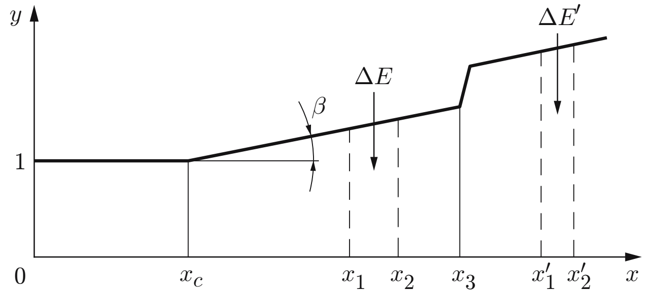

Fig. 8. Possible configuration of the channel in the case of energy supply in several isolated zones.

velocity being retained everywhere, including the period of the maximum energy supply. A possible configuration of the channel is shown in Fig. 8. The expansion angle should be large at the point of contour inflection x_3, which allows the total length of the channel to be reduced. To find the distribution of the amounts of energy supply over the sections, we can use the fact that gas heating at Strouchal numbers Sh $\gg 1$ is performed in small portions without formation of shock waves and does not exert any upstream influence. Therefore, the approximate solution can be obtained by solving a system of the form (1), where the last equation should be replaced by the equation for the entropy increment:

$$\frac{dS}{R} = \frac{\gamma}{\gamma - 1}\,\frac{dT}{T} - \frac{dp}{p} = \frac{H'(x) + H_d(x)}{RT}\,dx = \frac{1}{\gamma - 1}\,\frac{dT}{T} + \frac{H_d(x)}{RT}\,dx.$$

Integration is continued until the condition $T(x) = T_{\max}$ is satisfied or the entire specified amount of energy is supplied. If the condition $T(x) = T_{\max}$ is satisfied, the transition to the next section of energy supply occurs. The energy is not supplied in sections with drastic expansion of the channel: $H'(x) = 0$. The degree of expansion of these sections is either prescribed or determined from the Mach number specified at the end of the expanding section.

Conclusions. Pulsed-periodic supply of energy into a supersonic flow in a channel is the most effective method for energy addition, because the maximum efficiency of the gas is reached. A frequency of “saturation” of the energy pulses is obtained, above which the structure and the period-averaged distributions of parameters of the periodic flow remain unchanged. The frequency depends weakly on the problem parameters. By varying the sizes and locations of the energy-supply zones with a fixed power, one can control the flow structure. A channel configuration is proposed, which allows finding the distribution of the energy supplied from the condition of a restricted maximum value of the gas temperature. The simulation results presented show that pulsed-periodic supply of energy allows the flight Mach number to be increased to values that allow the effective specific impulse to be increased with the use of the ramjet channel within a combined engine.

REFERENCES

1. A. Ferri, “Review of SCRAMJET propulsion technology,” *J. Aircraft*, **5**, No. 1, 3–10 (1968).
2. A. Ferri and H. Fox, “Analysis of fluid dynamics of supersonic combustion process controlled by mixing,” in: *Abstr. of the 12th Intern. Symp. on Combustion*, Combustion Inst., Pittsburgh (1969), pp. 198–200.
3. V. P. Zamuraev, A. P. Kalinina, and A. F. Latypov, “Estimation of ramjet thrust under impulse energy release,” *Thermophys. Aeromech.*, **9**, No. 3, 385–390 (2002).
4. S. K. Godunov, A. V. Zabrodin, M. Ya. Ivanov, et al., *Numerical Solution of Multidimensional Problems in Gas Dynamics* [in Russian], Nauka, Moscow (1976).
5. R. W. Mackormack, “Numerical solution of the interaction of a shock wave with a laminar boundary layer,” *Lecture Notes Phys.*, **8**, 151–163 (1971).